Discrete Mathematics, MATH 132, Final, May 2005, Bilkent University

Time allowed: 110 minutes.

Attempt FIVE questions.

Each question is worth 20% .

Note: As usual, all graphs are understood to have only finitely many vertices, at most one edge between any two vertices; there are no loops, and all edges are undirected.

1: Find the generating function for the recurrence relation

$$a_{n+2} = a_{n+1} + a_n + \frac{1}{n!}$$

with initial conditions $a_0 = a_1 = 0$. (Express your answer in a simple form, without any infinite sums. You are NOT required to solve the recurrence relation.)

2: For each integer $n \geq 0$, let s_n be the number of n-digit sequences where each digit is 0 or 1 or 2, and there are no consecutive 0 digits. (For example, when $n = 2$, there are eight possible sequences: 01, 02, 10, 11, 12, 20, 21, 22.) Find a formula for s_n in terms of n.

3: For an integer $n \geq 2$, the graph K_n is the graph with n vertices where any two distinct vertices are connected by an edge.

(a) For which values of n does K_n have an Euler circuit?

(b) For which values of n does K_n have an Euler path which is not an Euler circuit.

(c) Repeat the question for the graph that is obtained from K_n by deleting one edge.

4: Show that, for any graph with at least two vertices, there exist two different vertices x and y which have the same degree as each other.

5: Let G be a connected planar graph with v vertices. Let d be a positive integer and suppose that every vertex of G has degree d. Show that v and d cannot both be odd.