Date: May 3, 2005
Time: 18:00–20:00

NAME: _____

STUDENT ID: _____

DEPARTMENT: CS   EE   IE

MATH 132 – MIDTERM 1

VERY IMPORTANT
1. Indicate clearly and unambiguously your final result. In proofs, state explicitly each claim.
2. Do not misread the questions or skip parts thereof. If you did, do not complain.
3. If you believe that a problem is misstated, do not solve it; explain your point of view instead.
4. Each problem has a reasonably short solution. If your calculation gets out of hand, something must be wrong.

TERMS AND CONDITIONS
1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
4. Calculators are <u>not</u> allowed.

Please do <u>not</u> write anything below this line.

1	2	3	4	5	TOTAL
20	20	20	20	20	100

1. If eleven integers are selected from $S = \{1, 2, 3, \ldots, 100\}$, prove that there are at least two, say x and y, such that $0 < \left| \sqrt{y} - \sqrt{x} \right| < 1$.

2. Let A be a set of size 3. How many partial orders on A are there such that:

1. Every element of A is minimal or maximal?

2. Every element of A is either minimal or maximal, but not both?

3. Let $A = \{1, 2, 3, \ldots, 10\}$ and $B = \{1, 2, 3, \ldots, 7\}$. How many functions $f \colon A \to B$ satisfy $|f(A)| = 4$? How many have $|f(A)| \leq 4$?

4. Find the generating function for the number of integer solutions of

1. $2w + 3x + 5y + 7z = n, \quad 0 \leq w, x, y, z$;

2. $2w + 3x + 5y + 7z = n, \quad 0 \leq w, \quad 4 \leq x, y, \quad 5 \leq z$.

5. How many 20 digit sequences made out of 0, 1, 2, 3 are there where:

1. There is at least one 2 and an odd number of 0's?

2. No digit occurs exactly twice?

3. No digit occurs exactly three times?

4. There are no 3's or exactly two 3's?