ODTU-Bilkent Algebraic Geometry

Lines on singular quartic surfaces via Vinberg

By

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Abstract: Large configurations of lines (or, more generally, rational curves of low degree) on algebraic surfaces appear in various contexts, but only in the case of cubic surfaces the picture is complete. Our principal goal is the classification of large configurations of lines on quasi-polarized K3-surfaces in the presence of singularities. To the best of our knowledge, no attempt has been made to attack this problem from the lattice-theoretical, based on the global Torelli theorem, point of view; some partial results were obtained by various authors using ``classical'' algebraic geometry, but very little is known. The difficulty is that, given a polarized N\'eron--Severi lattice, computing the classes of smooth rational curves depends on the choice of a Weyl chamber of a certain root lattice, which is not unique.
We show that this ambiguity disappears and the algorithm becomes deterministic provided that sufficiently many classes of lines are fixed. Based on this fact, Vinberg's algorithm, and a combinatorial version of elliptic pencils, we develop an algorithm that, in principle, would list all extended Fano graphs. After testing it on octic K3-surfaces, we turn to the most classical case of simple quartics where, prior to our work, only an upper bound of 64 lines (Veniani, same as in the smooth case) and an example of 52 lines (the speaker) were known. We show that, in the presence of singularities, the sharp upper bound is indeed 52, substantiating the long standing conjecture (by the speaker) that the upper bound is reduced by the presence of smooth rational curves of lower degree.
We also extend the classification (I. Itenberg, A.S. Sertöz, and the speaker) of large configurations of lines on smooth quartics down to 49 lines. Remarkably, most of these configurations were known before.
This project was conceived and partially completed during our joint stay at the Max-Planck-Institut f\ür Mathematik, Bonn. The speaker is partially supported by TÜBİTAK project 123F111.

Date: 15 December 2023, Friday
Time: 15:40 (GMT+3)
Place: Zoom

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