



TOPOLOGY SEMINAR

Advances on Quillen's conjecture

By

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Abstract: The study of the p -subgroup complexes began motivated by group cohomology and equivariant cohomology of topological spaces "modulo the prime p ". For example, Kenneth Brown proved that the reduced Euler characteristic of this complex is divisible by the size of a Sylow p -subgroup, giving rise to a sort of "Homological Sylow theorem". Later, he showed that the mod- p equivariant cohomology of the p -subgroup complex of a finite group coincides with the mod- p cohomology of the group. Deeper relations with finite group theory, representation theory, and finite geometries were also explored. For instance, uniqueness of certain simple groups, finite geometries for sporadic groups, Lefschetz modules, and, more recently, endotrivial modules.

In 1978, Daniel Quillen conjectured that the poset of non-trivial p -subgroups of a finite group G is contractible if and only if G has non-trivial p -core. Quillen established the conjecture for solvable groups and some families of groups of Lie type. The major step towards the resolution of the conjecture was done by Michael Aschbacher and Stephen D. Smith at the beginning of the nineties. They roughly proved that if $p > 5$ and G is a group of minimal order failing the conjecture, then G contains a simple component $PSU(n, q^2)$ failing a certain homological condition denoted by (QD) (namely, the top-degree homology group of its p -subgroup poset does not vanish).

In this talk, I will present recent advances in the conjecture, with a particular focus on the prime $p=2$, which was not covered by the methods developed by Aschbacher-Smith. In particular, we show that the study of the conjecture for the prime $p=2$ basically reduces to studying (QD) on the poset of p -subgroups of certain families of classical groups. Part of this work is in collaboration with S.D. Smith

Date: Monday, December 4, 2023

Time: 13:30

Place: ZOOM

To request the event link, please send a message to cihan.okay@bilkent.edu.tr