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Problem Of The Month

Term: November 2018

A point with the coordinates (a, b) on the plane is a *primitive point* if a, b are integers with $\gcd(a, b) = 1$. A graph whose vertices are primitive points is constructed as follows: an edge is drawn between points (a_1, b_1) and (a_2, b_2) if and only if $2a_1 = 2a_2 \in \{b_1 - b_2, b_2 - b_1\}$ or $2b_1 = 2b_2 \in \{a_1 - a_2, a_2 - a_1\}$. Later, some edges of this graph will be removed until a forest is obtained. At least how many edges must be removed from the graph? At least how many trees will be found in the forest?

Note: A forest is a union of disjoint trees.