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Problem Of The Month

May 2015

Problem: We say that a triple (u_1, u_2, u_3) of unit squares of 2015×2015 grid is *L-shaped* if u_1 and u_2 lie on the same column, u_1 is above u_2, u_3 shares the same line with u_2 and lies to the right of u_2. Find the minimal possible value of k if each unit square of 2015×2015 grid is colored into one of k colors so that no L-shaped triple is mono colored.

Solution: The answer: $k = 1008$.

Let us prove that are at most 4029 unit squares colored in any particular color, say red. Indeed, let us mark the rightmost red unit square in each line. Then each column (except the last one which does not contain any unmarked red square) contains at most one unmarked red unit square since otherwise there is a L-shaped triple (u_1, u_2, u_3) colored red. Therefore, there are at most 2015 marked and 2014 unmarked, in total 4029 unit red squares. Thus, the total number of colors $k \geq \frac{2015 \cdot 2015}{4029} > 1007$. A coloring for 1008 colors: let us numerate lines from top to bottom and columns from left to right by $1, 2, \ldots, 2015$ and color the unit square (i, j) lying in the intersection of i-th line and j-th column by the color $\lfloor \frac{(i+j)(mod 2016)}{2} \rfloor$.