



Bilkent University
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PROBLEM OF THE MONTH

October 2012

Problem:

There are n points in the space, some pairs of points are connected by line segments. Suppose that the total number of line segments is 3018 and each line segment is colored in one of the given three colors so that after removing of all line segments of *any color* each pair of points will be connected by some path consisting of line segments. Find the maximal possible value of n .

Solution:

Assume that x_1 segments are colored 1, x_2 segments are colored 2, x_3 segments are colored 3 and $x_1 \leq x_2 \leq x_3$. If segments colored 3 are removed the number of remaining segments is at most 2012. Since any connected graph with m vertices has at least $m - 1$ edges we conclude that $n \leq 2013$.

Below we give an example for $n = 2013$:

Let us place 2013 points on $x-y$ plane with coordinates $(1, 0), (2, 0), \dots, (1006, 0), (1007, 0), (1, 1), (2, 1), \dots, (1006, 1)$ and connect

pairs $(i, 0)$ and $(i, 1)$ $i = 1, 2, \dots, 1006$ by color 1;

pairs $(i, 0)$ and $(i + 1, 0)$ $i = 1, 2, \dots, 1006$ by color 2;

pairs $(i, 1)$ and $(i + 1, 1)$ $i = 1, 2, \dots, 1005$ and pair $(1006, 1)$ and $(1007, 0)$ by color 3.

It can be easily seen that this coloring satisfies the conditions. The answer: $n = 2013$.