Bilkent University
Department of Mathematics

Problem Of The Month

October 2010

Problem:

Suppose that a real polynomial P of degree 2010 has 2010 distinct real roots. Let $q(P)$ be the total number of nonzero coefficients of P. What is the minimal possible value of $q(P)$?

Solution:

Between arbitrary two real roots of a polynomial P there is a real root of the polynomial P'. Therefore, P' has $n-1$ distinct real roots. By repeating of this argument we get that k-th derivative polynomial P^k has $2010-k$ distinct real roots. Let a_l and a_{l-1} be two neighboring coefficients of the polynomial P. Let us show that at least one of them is not zero. On the contrary, if $a_l = a_{l-1} = 0$ then the last two coefficients (the coefficient at x and x^0) of the polynomial P^l are zeros and P^l has a multiple root 0, which contradicts to the fact that P^l has $2010-l$ distinct real roots. Therefore, at least 1005 coefficients of P are non-zeros and $q(P) \geq 1006$. The polynomial $P(x) = \prod_{s=1}^{1005}(x^2 - s^2)$ of degree 2010 has 2010 distinct real roots and 1006 nonzero coefficients. Thus, $q(P) = 1006$.