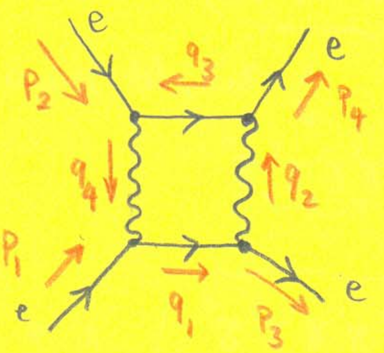
Phys 453, $2^{nd}$ Midterm Solutions 13 April 2012

1]




external & internal momenta are labelled

$$\iiiint \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}\frac{d^4q_4}{(2\pi)^4}\;\bar{u}(4)\,\gamma^\mu\,\frac{i(\not{q}_3+mc)}{q_3^2-m^2c^2}\,\gamma^\nu u(2)\;\frac{-ig^{\mu\lambda}}{q_2^2}\; \hookleftarrow$$

$$\hookrightarrow\;\bar{u}(3)\,\gamma^\lambda\,\frac{i(\not{q}_1+mc)}{q_1^2-m^2c^2}\,\gamma^\sigma u(1)\;\frac{-ig^{\nu\sigma}}{q_4^2}\cdot\left(ig_e\right)^4 \hookleftarrow$$

$$\hookrightarrow\left[(2\pi)^4\right]^4\;\underbrace{\delta^4(p_1-q_1+q_4)}_{q_1\to p_1+q_4}\;\underbrace{\delta^4(-p_3+q_1-q_2)}_{q_2\to \underbrace{q_1-p_3}}\;\underbrace{\delta^4(-p_4+q_2-q_3)}_{q_3\to \underbrace{q_2-q_4}}\;\underbrace{\delta^4(p_2+q_3-q_4)}_{\text{First replace } q_3 \text{ by } p_1-p_3+q_4-p_4}$$

$q_2 \to p_1-p_3+q_4$ $\quad q_3\to p_1-p_3+q_4-p_4$ $\quad \underbrace{\delta^4(p_1+p_2-p_3-p_4)}_{\text{Replace w/ } i/(2\pi)^4}$

Relabel: $q_4\to q$ and $\gamma^\mu g^{\mu\lambda}\gamma^\lambda = \gamma^\mu\gamma_\mu$ , $\gamma^\nu g^{\nu\sigma}\gamma^\sigma = \gamma^\nu\gamma_\nu$

$$\Rightarrow \mathcal{M} = \int \frac{d^4q}{(2\pi)^4}\; ig_e^4\left[\bar{u}(4)\gamma^\mu\gamma^\nu u(2)\right]\left[\bar{u}(3)\gamma_\mu\gamma_\nu u(1)\right]\frac{\not{p}_1-\not{p}_3-\not{p}_4+\not{q}+mc}{(p_1-p_3-p_4+q)^2-m^2c^2}\;\hookleftarrow$$

$$\hookrightarrow\;\frac{\not{p}_1+\not{q}+mc}{(p_1+q)^2-m^2c^2}\quad\frac{1}{(p_1-p_3+q)^2}\quad\frac{1}{q^2}$$

2] $\{\gamma^5, \gamma^\mu\} = i\left(\gamma^0\gamma^1\gamma^2\gamma^3\gamma^\mu + \gamma^\mu\gamma^0\gamma^1\gamma^2\gamma^3\right)$

First, let $\mu = 0$

$$\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0$$

$$-1 \quad -1 \quad -1$$

$$i\left(\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 - \gamma^0\gamma^1\gamma^2\gamma^3\gamma^0\right) \equiv 0$$

Next if we do this for $\mu = 1, 2, 3$ we get the same result, because flipping of $\gamma^\mu$ in $\gamma^\mu\gamma^0\gamma^1\gamma^2\gamma^3$ always meets a $\gamma^\mu$, resulting in an overall $-$ sign.

Hence $\{\gamma^5, \gamma^\mu\} = 0$ for $\mu = 0, 1, 2, 3$

3] This is almost trivial.

$$\not{p}\not{p} = \gamma^\mu p_\mu \gamma^\nu p_\nu = p_\mu p_\nu \gamma^\mu\gamma^\nu$$

Use $\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu + 2g^{\mu\nu}$

$$\underbrace{p_\mu p_\nu \gamma^\mu\gamma^\nu + p_\mu p_\nu \gamma^\nu\gamma^\mu}_{2\not{p}\not{p}} = 2\underbrace{g^{\mu\nu} p_\mu p_\nu}_{p^\mu p_\mu}$$

$$2\not{p}\not{p} = 2p^\mu p_\mu$$

$$\therefore \quad \not{p}\not{p} = p^2$$