Phys 453, 1st Midterm Solutions 9 March 2012

1] a)

$E_{L,tot} = (1+\gamma) mc^2$ $\qquad$ $E_{CM,tot} = 2\gamma' mc^2$

$P_{L,tot} = \gamma m u$ $\qquad$ $P_{CM,tot} = 0$

Using the invariance of $P_\mu P^\mu$ among frames

$$(\gamma+1)^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = 4\gamma'^2 m^2 c^4$$

$$\Rightarrow \quad \gamma'^2 = \frac{(\gamma+1)^2 - \gamma^2 \underbrace{u^2/c^2}_{\beta^2}}{4} = \frac{\underbrace{\gamma^2 + 2\gamma + 1 - \gamma^2\beta^2}_{\gamma^2 - \gamma^2\beta^2 = 1}}{4} = \frac{\gamma+1}{2}$$

$$\gamma'^2 = \frac{1}{1-\beta'^2}, \qquad \beta'^2 = (u'/c)^2$$

$$\Rightarrow \quad 1 - \left(\frac{u'}{c}\right)^2 = \frac{2}{1+\gamma} \quad \text{or} \quad \frac{u'}{c} = \sqrt{\frac{\gamma-1}{\gamma+1}}$$

$$\Rightarrow \quad u' = c\,\frac{\gamma-1}{\sqrt{\gamma^2-1}} = c\,\frac{\gamma-1}{\beta\gamma} = \frac{c}{\beta}\left(1-\frac{1}{\gamma}\right) = \frac{c^2}{u}\left[1-\sqrt{1-\frac{u^2}{c^2}}\right]$$

b) For $u \ll c$ we use $\sqrt{1-\frac{u^2}{c^2}} \simeq 1 - \frac{u^2}{2c^2}$

$$\Rightarrow \quad u' \xrightarrow{NR} \frac{c^2}{u}\left[1-\left(1-\frac{u^2}{2c^2}\right)\right] = \frac{u}{2}$$ ... non-relativistic value in the CM frame

NB: An alternative solution is to make use of velocity addition rule.

2]

$$\underset{M}{\bullet} \qquad \overset{m-\delta m}{\longleftarrow\bullet} \rightsquigarrow h\nu$$

$E_T = mc^2$

$P_T = 0$

Before

$E_T' = h\nu + \gamma'(m-\delta m)c^2 = mc^2 \cdots$ cons. of en.

$P_T' = 0$

After

For the atom after radiation, we can write:

$$\underbrace{\gamma'^2 (m-\delta m)^2 c^4}_{(mc^2 - h\nu)^2} - (h\nu)^2 = (m-\delta m)^2 c^4$$

$$\Rightarrow \cancel{m^2 c^4} - 2h\nu mc^2 = [\cancel{m^2} - 2\delta m\, m - \delta m^2]\, c^4$$

$$\Rightarrow \quad h\nu = c^2 \delta m \left(1 - \frac{\delta m}{2m}\right)$$

3] This is the addition of 1 and 1/2.

a) $J = 1 + \frac{1}{2} = \frac{3}{2} \Rightarrow \{|J,M\rangle\} = \{|\frac{3}{2}\ \frac{-3}{2}\rangle, |\frac{3}{2}\ \frac{-1}{2}\rangle, |\frac{3}{2}\ \frac{1}{2}\rangle, |\frac{3}{2}\ \frac{3}{2}\rangle\}$

$J = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \{|J,M\rangle\} = \{|\frac{1}{2}\ \frac{-1}{2}\rangle, |\frac{1}{2}\ \frac{1}{2}\rangle\}$

b) Given that $J = 1/2$ and $m = -1/2$

Using the C-G table we get:

$$|\tfrac{1}{2}\ \tfrac{-1}{2}\rangle = \frac{1}{\sqrt{3}} |0\ \tfrac{1}{2}\rangle - \sqrt{\frac{2}{3}}\, |-1\ \tfrac{1}{2}\rangle$$

$\Rightarrow \quad L_z = 0$ with probability 1/3 and -1 with probability 2/3