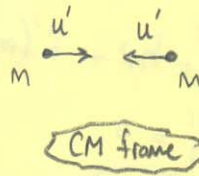
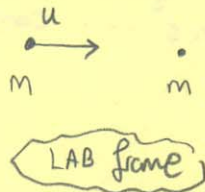


# Phys 453, 1<sup>st</sup> Midterm Solutions

9 March 2012

1]

a)



$$E_{L,tot} = (1+\gamma)mc^2$$

$$E_{CM,tot} = 2\gamma' mc^2$$

$$P_{L,tot} = \gamma mu$$

$$P_{CM,tot} = 0$$

Using the invariance of  $P_\mu P^\mu$  among frames.

$$(\gamma+1)^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = 4\gamma'^2 m^2 c^4$$

$$\Rightarrow \gamma'^2 = \frac{(\gamma+1)^2 - \gamma^2 \frac{u^2}{c^2}}{4} = \frac{\gamma^2 + 2\gamma + 1 - \gamma^2 \beta^2}{4} = \frac{\gamma+1}{2}$$

$$\frac{1}{1-\beta'^2} \left(\frac{u'}{c}\right)^2 \Rightarrow 1 - \left(\frac{u'}{c}\right)^2 = \frac{2}{1+\gamma} \quad \text{or} \quad \frac{u'}{c} = \sqrt{\frac{\gamma-1}{\gamma+1}}$$

$$\Rightarrow u' = c \frac{\gamma-1}{\sqrt{\gamma^2-1}} = c \frac{\gamma-1}{\beta\gamma} = \frac{c}{\beta} \left(1 - \frac{1}{\gamma}\right) = \frac{c^2}{u} \left[1 - \sqrt{1 - \frac{u^2}{c^2}}\right]$$

b) For  $u \ll c$  we use  $\sqrt{1 - \frac{u^2}{c^2}} \approx 1 - \frac{u^2}{2c^2}$

$$\Rightarrow u' \xrightarrow{NR} \frac{c^2}{u} \left[1 - \left(1 - \frac{u^2}{2c^2}\right)\right] = \frac{u}{2} \dots \text{non-relativistic value in the CM frame}$$

NB: An alternative solution is to make use of velocity addition rule.

2]

$m$   
 $\bullet$   
 $E_T = mc^2$   
 $P_T = 0$   
Before

$m - \delta m$   
 $\leftarrow \bullet \rightsquigarrow h\nu$   
 $E'_T = h\nu + \gamma'(m - \delta m)c^2 = mc^2 \dots$  cons. of en.  
 $P'_T = 0$   
After

For the atom after radiation, we can write:

$$\underbrace{\gamma'^2 (m - \delta m)^2 c^4}_{(mc^2 - h\nu)^2} - (h\nu)^2 = (m - \delta m)^2 c^4$$

$$\Rightarrow \cancel{m^2 c^4} - 2h\nu m c^2 = [m^2 - 2\delta m m - \delta m^2] c^4$$

$$\Rightarrow h\nu = c^2 \delta m \left(1 - \frac{\delta m}{2m}\right)$$

3] This is the addition of 1 and  $\frac{1}{2}$ .

$$a) \quad J = 1 + \frac{1}{2} = \frac{3}{2} \Rightarrow \{|J, M\rangle\} = \left\{ \left| \frac{3}{2}, \frac{3}{2} \right\rangle, \left| \frac{3}{2}, \frac{1}{2} \right\rangle, \left| \frac{3}{2}, \frac{-1}{2} \right\rangle, \left| \frac{3}{2}, \frac{-3}{2} \right\rangle \right\}$$

$$J = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \{|J, M\rangle\} = \left\{ \left| \frac{1}{2}, \frac{1}{2} \right\rangle, \left| \frac{1}{2}, \frac{-1}{2} \right\rangle \right\}$$

b) Given that  $J = \frac{1}{2}$  and  $m = -\frac{1}{2}$

Using the C-G table we get:

$$\left| \frac{1}{2}, \frac{-1}{2} \right\rangle = \frac{1}{\sqrt{3}} \left| 0, \frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \left| -1, \frac{1}{2} \right\rangle$$

$\Rightarrow L_z = 0$  with probability  $\frac{1}{3}$  and  $-1$  with probability  $\frac{2}{3}$