Practice Set-1

Phys 438/538: Atomic, Molecular and Optical Physics

11 February 2016

Not to be graded

Solve the following three problems (1.2, 1.3, 1.4) from Ch. 1 of Hill & Lee.

Their images are given below (Note the correction in Eq. 1.105):

1.2 Calculate

$$\langle r^{\gamma} \rangle = \int r^{2+\gamma} R_{nl}^{2}(r)\, dr \tag{1.100}$$

for $\gamma = -3, -2, -1, 1, 2, 3$.

1.3 Since the wavefunctions of the s levels are finite at the origin, the potential for s electrons changes when the electron penetrates the nucleus and gives rise to a shift in the energy levels relative to Eq. (1.42). If the radius of the nucleus is r_o, the potential will be given by

$$\begin{aligned} &\frac{Q}{|r|} && \text{when} \quad r > r_o, \\ &\int_N \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r' && \text{when} \quad r < r_o, \end{aligned} \tag{1.101}$$

where ρ is the charge density of the nucleus. Use first-order perturbation theory to show that the s states are shifted by

$$(\Delta E_s)_{\text{finite size}} = \frac{2}{3} \frac{e^2}{4\pi\varepsilon_o} \frac{\langle r^2 \rangle_N}{a_o^3}, \tag{1.102}$$

where

$$\left\langle r^2 \right\rangle_N = \int_N r'^2 \rho(\vec{r}')\, d^3 r'. \tag{1.103}$$

If you take the size of the nucleus to be about 8 fm, estimate the magnitude of this shift for the ground state, i.e., calculate $(\Delta E_s)_{\text{finite size}} / \mathrm{E}_{1s}$.

1.4 Show that

$$\frac{e\hbar}{4m_e^2 c^2} \underline{\sigma} \cdot \vec{\nabla}\phi \times \vec{p} \tag{1.104}$$

in Table 1.1 can be rewritten as

$$\frac{e\hbar^2}{2m_e^2 c^2} \frac{1}{r} \frac{d\phi}{dr} \left(\vec{L} \cdot \vec{S} \right). \tag{1.105}$$