Phys 438/538 Atomic, Molecular and Optical Physics

First Midterm Examination

8 March 2016

Duration: 110 Minutes, Closed notes/books

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1) (25 points) Radial expectations via Hellmann-Feynman theorem
For radial expectations values, a short-cut alternative is to make use of the Hellmann-Feynman theorem, which states that if the Hamiltonian depends on a continuous parameter λ, then

$$\langle \gamma, \lambda | \frac{\partial H(\lambda)}{\partial \lambda} | \gamma, \lambda \rangle = \frac{\partial E_\gamma(\lambda)}{\partial \lambda} ,$$

where γ designates a group of quantum labels, and $E_\gamma(\lambda)$ satisfies the eigenvalue equation $H(\lambda)|\gamma, \lambda\rangle = E_\gamma(\lambda)|\gamma, \lambda\rangle$. You will apply it to the one-electron H-like ion radial Hamiltonian

$$H_r = -\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2}\right] - \frac{Ze^2}{4\pi\epsilon_0 r} ,$$

having $E_{k\ell}(\lambda) = -\frac{1}{2(k+\ell+1)^2}\frac{Z^2\mu e^4}{(4\pi\epsilon_0\hbar)^2}$, where μ is the reduced mass of the ion, and $n = k + \ell + 1$ is the well-known principal quantum number. Using Hellmann-Feynman theorem, find out the radial expectations $\langle \frac{1}{r} \rangle_{n\ell}$ and $\langle \frac{1}{r^2} \rangle_{n\ell}$. Express your results in terms of Bohr radius, $a_B = \frac{4\pi\epsilon_0\hbar^2}{e^2 m_e}$ to clearly show its dimensional correctness.

2) (30 points) Spin-orbit Interaction
In the one-electron case, the spin-orbit interaction term is given as

$$H_{SO} = \frac{1}{2m_e^2c^2}\frac{Ze^2}{4\pi\epsilon_0 r^3}\left(\vec{\ell}\cdot\vec{s}\right) .$$

Considering sodium atom with atomic number 11, determine the energy of the splitting due to spin-orbit interaction for the case when the valence electron is *excited* to $5p$ level: Treat it as a hydrogen-like ion and put your final result in the form

$$\Delta E_{SO}(5p) = A\frac{1}{4\pi\epsilon_0}\frac{e^2\hbar^2}{m_e^2c^2a_B^3} ,$$

and give the numerical value of the coefficient A. Note that $\langle \frac{1}{r^3} \rangle_{n\ell} = \frac{Z^3}{a_B^3 n^3(\ell+1)(\ell+1/2)\ell}$, which can actually be calculated following the recipe in question 1 (but for the interest of time, use directly this result).

3) (20 points) Angular momentum coupling and Hund's Rules
(a) Scandium has a valence $3d^1$ electron. For the following microstates in orbital and spin angular momentum basis $|m_l = 0, m_s = -1/2\rangle$, $|m_l = 1, m_s = -1/2\rangle$, what are the underlying $|J, m_J\rangle$ microstates states in total angular momentum basis, and their relative probabilities in each case?
(b) Find the ground state spectroscopic term symbols for Sm^{15+} [Xe] $4f$, and Ir^{17+} [Xe] $4f^{13}\,5s$.

4) (25 points) Zeeman Splitting
For the ground states of Pr^{3+} [Xe] $4f^2$, and Eu^{3+} [Xe] $4f^6$ work out the and sketch the shifts of all magnetic sublevels as a function of external magnetic field, B. Note that for both cases LS coupling holds, and Landé g-factor, $g_J = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$.

43. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	…
		M	M	…
m_1	m_2	Coefficients		
m_1	m_2			
.	.			
.	.			
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1	m_2	1, +1
+1/2	+1/2	1

m_1	m_2	1, 0	0, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1, −1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2, +3/2
+1	+1/2	1

m_1	m_2	3/2, +1/2	1/2, +1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2, −1/2	1/2, −1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2, −3/2
−1	−1/2	1

2×1

m_1	m_2	3, +3
+2	+1	1

m_1	m_2	3, +2	2, +2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3, +1	2, +1	1, +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3, 0	2, 0	1, 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3, −1	2, −1	1, −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3, −2	2, −2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3, −3
−2	−1	1

1×1

m_1	m_2	2, +2
+1	+1	1

m_1	m_2	2, +1	1, +1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2, 0	1, 0	0, 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2, −1	1, −1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2, −2
−1	−1	1

2×1/2

m_1	m_2	5/2, +5/2
+2	+1/2	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2, +1/2	3/2, +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2, −1/2	3/2, −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2, −3/2	3/2, −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2, −5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	2, +2
+3/2	+1/2	1

m_1	m_2	2, +1	1, +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2, 0	1, 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2, −1	1, −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2, −2
−3/2	−1/2	1

3/2×1

m_1	m_2	5/2, +5/2
+3/2	+1	1

m_1	m_2	5/2, +3/2	3/2, +3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2, −3/2	3/2, −3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2, −5/2
−3/2	−1	1

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

3/2×3/2

m_1	m_2	3, +3
+3/2	+3/2	1

m_1	m_2	3, +2	2, +2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	3, +1	2, +1	1, +1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	3, 0	2, 0	1, 0	0, 0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	3, −1	2, −1	1, −1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	3, −2	2, −2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	3, −3
−3/2	−3/2	1

2×3/2

m_1	m_2	7/2, +7/2
+2	+3/2	1

m_1	m_2	7/2, +5/2	5/2, +5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	7/2, −1/2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	7/2, −3/2	5/2, −3/2	3/2, −3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	7/2, −5/2	5/2, −5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	7/2, −7/2
−2	−3/2	1

2×2

m_1	m_2	4, +4
+2	+2	1

m_1	m_2	4, +3	3, +3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	4, +2	3, +2	2, +2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	4, +1	3, +1	2, +1	1, +1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	4, −1	3, −1	2, −1	1, −1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	4, −2	3, −2	2, −2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	4, −3	3, −3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	4, −4
−2	−2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 43.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).

Hellmann - Feynman Theorem

When a parameter λ of the Hamiltonian is (or promoted to being) a continuous variable, then for $H(\lambda)|\gamma,\lambda\rangle = E_\gamma(\lambda)|\gamma,\lambda\rangle$

we have
$$\langle\gamma,\lambda|\frac{\partial H(\lambda)}{\partial\lambda}|\gamma,\lambda\rangle = \frac{\partial E_\gamma(\lambda)}{\partial\lambda}$$

Proof: Based on the normalization of eigenkets: $\langle\gamma,\lambda|\gamma,\lambda\rangle = 1$

So, if we differentiate $\langle\gamma,\lambda|H(\lambda)|\gamma,\lambda\rangle = E_\gamma(\lambda)$ wrt λ

$$\left(\frac{\partial}{\partial\lambda}\langle\gamma,\lambda|\right)\overbrace{H(\lambda)|\gamma,\lambda\rangle}^{E_\gamma(\lambda)|\gamma,\lambda\rangle} + \overbrace{\langle\gamma,\lambda|H(\lambda)}^{E_\gamma(\lambda)\langle\gamma,\lambda|}\frac{\partial}{\partial\lambda}|\gamma,\lambda\rangle +$$

$$\hookrightarrow \langle\gamma,\lambda|\frac{\partial H(\lambda)}{\partial\lambda}|\gamma,\lambda\rangle = \frac{\partial E_\gamma(\lambda)}{\partial\lambda}$$

$$\therefore\quad \underbrace{E_\gamma(\lambda)\frac{\partial}{\partial\lambda}\Big(\underbrace{\langle\gamma,\lambda|\gamma,\lambda\rangle}_{1\ (\text{indep. of } \lambda)}\Big)}_{0} + \langle\gamma,\lambda|\frac{\partial H(\lambda)}{\partial\lambda}|\gamma,\lambda\rangle = \frac{\partial E_\gamma(\lambda)}{\partial\lambda}$$

QED

The Hellmann-Feynman theorem, when applicable greatly simplifies the radial expectations, turning the labor to simple derivatives!

The radial part of the H-like ion is given by

$$H_r = -\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right] - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

with the corresponding eigenvalue $E_{kl}(\lambda) = -\frac{1}{2(k+l+1)^2}\frac{Z^2\mu e^4}{(4\pi\varepsilon_0\hbar)^2}$

where $k \in \mathbb{Z}$ that determines when the series obtained in the solution of the radial eqn. terminates (see D.J. Griffiths IQM),

and $n = k + l + 1$ is the well-known principal quantum number

∗ For $\left\langle \frac{1}{r} \right\rangle_{nl}$, we can promote Z to a 'fake' continuous var.

$$\Rightarrow \lambda = Z \quad \text{so that} \quad -\frac{4\pi\varepsilon_0}{e^2}\frac{\partial}{\partial Z} H_r(Z) = \frac{1}{r}$$

So, we only need to work out the derivative $-\frac{4\pi\varepsilon_0}{e^2}\frac{\partial}{\partial Z} E_{kl}(Z)$

$$\frac{4\pi\varepsilon_0}{e^2}\frac{1}{2(\underbrace{k+l+1}_{n})^2}\frac{2Z\mu e^4}{(4\pi\varepsilon_0\hbar)^2} = \frac{Z}{n^2}\underbrace{\left(\frac{e^2 m_e}{4\pi\varepsilon_0\hbar^2}\right)}_{\frac{1}{a_B}}\frac{\mu}{m_e}$$

$$\therefore \left\langle \frac{1}{r} \right\rangle_{nl} = \frac{Z}{a_B n^2}\cdot\frac{\mu}{m_e}$$

∗ For $\left\langle \frac{1}{r^2} \right\rangle_{nl}$ choose $l \to \lambda$ the "fake" cont. var.

$$\Rightarrow \left\langle \frac{1}{r^2} \right\rangle_{nl} = \left\langle -\frac{2\mu}{\hbar^2(2l+1)}\frac{\partial H_r}{\partial l} \right\rangle_{nl} \overset{H-F}{=} -\frac{2\mu}{\hbar^2(2l+1)}\frac{\partial}{\partial l}E_{kl}(l)$$

$$= \frac{2\mu}{\hbar^2(2l+1)}\frac{1}{2}\cdot\frac{Z^2\mu e^4}{(4\pi\varepsilon_0\hbar)^2} - 2\cdot(k+l+1)^{-3}$$

$$= \frac{Z^2\mu^2 e^4}{(4\pi\varepsilon_0\hbar^2)^2}\frac{1}{(l+\frac{1}{2})n^3} = \left(\frac{Z\mu}{m_e}\right)^2\frac{1}{n^3(l+\frac{1}{2})a_B^2}$$

2]

$$H_{so} = \frac{1}{2m_e^2c^2} \frac{Ze^2}{4\pi\varepsilon_0 r^2} \vec{l}\cdot\vec{s}$$

$\frac{1}{r}\frac{dU}{dr}$ term we had in the notes

p shell: $l=1$ $s=1/2$

Using $\underbrace{\vec{j} = \vec{l}+\vec{s}}$, $\vec{j}\cdot\vec{j} = \hat{j}^2 = l^2+s^2+2\,\vec{l}\cdot\vec{s}$

$\hat{j}_1 = \underbrace{l+s}_{3/2}$ $\hat{j}_2 = \underbrace{l-s}_{1/2}$ $\Rightarrow \vec{l}\cdot\vec{s} = \frac{1}{2}\left(j^2-l^2-s^2\right)$

Since $\langle j^2 \rangle = \hbar^2 j(j+1)$, so we can work out the energy splitting due to SOI bet. $j_1 = 3/2$ and $j_2 = 1/2$ states

$$\Delta E_{so} = \frac{Ze^2}{8\pi\varepsilon_0 m_e^2c^2} \frac{\hbar^2}{2}\left[j_1(j_1+1) - j_2(j_2+1)\right] \underbrace{\left\langle \frac{1}{r^3}\right\rangle}_{\frac{Z^3}{a_B^3 n^3 l(l+\frac{1}{2})(l+1)}}$$

given in problem

Putting together

$$\Delta E_{SOI} = A \frac{1}{4\pi\varepsilon_0}\frac{e^2\hbar^2}{m_e^2c^2a_B^3} \quad \text{where} \quad A = \frac{Z^4\left[j_1(j_1+1)-j_2(j_2+1)\right]}{4n^3 l(l+\frac{1}{2})(l+1)}$$

For $\underbrace{Z=11,\ l=1,\ n=5}_{\text{Sodium 5p}}$ $A = 29.28$

3] From C-G table for d-shell single $\bar{e}$: $l=2$, $s=1/2$

(a)

$|m_l, m_s\rangle$: $|0, -1/2\rangle \longrightarrow |\frac{5}{2}, -\frac{1}{2}\rangle$ state with pr. $\frac{3}{5}$ & $|\frac{3}{2}, -\frac{1}{2}\rangle$ with pr. $\frac{2}{5}$

$|1, -1/2\rangle \longrightarrow |\frac{5}{2}, \frac{1}{2}\rangle$ " " " $\frac{2}{5}$, $|\frac{3}{2}, \frac{1}{2}\rangle$ " " $\frac{3}{5}$

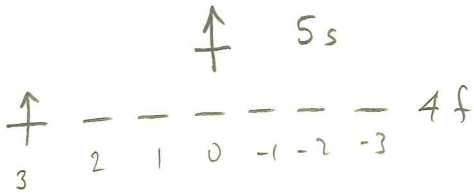
3b]

Sm^{15+} 4f $\xrightarrow[\text{state}]{\text{ground}}$

$\underbrace{L = 3}_{F}$, $S = 1/2$ → $J_{max} = 7/2$ → $J_{min} = 5/2$

Less than half-filled ⇒ $J \rightarrow J_{min}$ $^2F_{5/2}$

Ir^{17+} $4f^{13}\ 5s$



$\sum m_s = 1$

$\sum m_{\ell_i} = 3 \longrightarrow L = 3 \rightarrow F$, $S = 1$

$J_{min} = 2$, $J_{max} = 4$

> half filled (f shell dominates)

⇒ $J \rightarrow J_{max}$ $^3F_4$

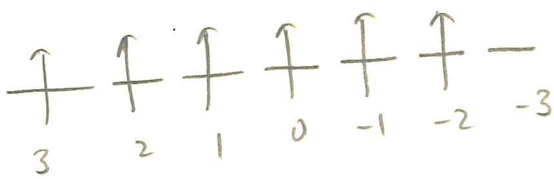
4]

Pr^{3+} $4f^2$

$S = 1$, $L = 5 \rightarrow H$ } $J_{max} = 6$, $J_{min} = 4$

< Half filled $^3H_4$

Eu^{3+} $4f^6$



$S = 3$

$L = \sum_i m_{\ell_i} = 3 \rightarrow F$

$J_{min} = 0$, $J_{max} = 6$

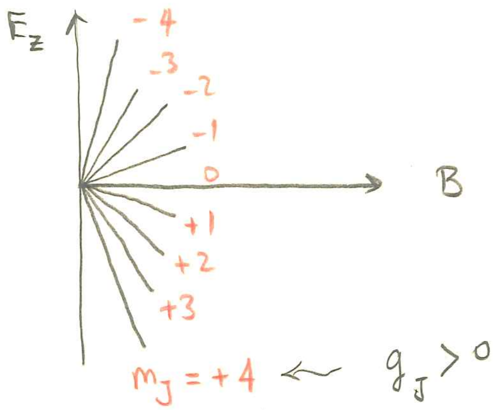
< Half-filled $^7F_0$

Zeeman Splittings

$Pr^{3+} \rightarrow {}^3H_4 \qquad S=1, \; L=5, \; J=4$

$$g_J = 1 + \frac{4\cdot5 + 1\cdot2 - 5\cdot6}{2\cdot4\cdot5} = \frac{4}{5}$$

$$E_Z = -\frac{4\mu_B B}{5} m_J \quad ; \quad m_J = \underbrace{0, \pm1, \pm2, \pm3, \pm4}_{9 \text{ Zeeman sublevels}}$$



$\leftarrow g_J > 0$

$Eu^{3+} \rightarrow {}^7F_0 \qquad J=0 \rightarrow$ No Zeeman splitting

$$g_J \equiv 0$$