Phys 438 Atomic and Molecular Physics
Phys 538 Light-Matter Interactions

First Midterm Examination

22 March 2013

Duration: Two hours, Closed notes/books

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1) (30 points) Relativistic Corrections
Find the first-order relativistic corrections due to (*i*) spin-orbit interaction, (*ii*) relativistic mass correction, and (*iii*) Darwin terms, for the $2s$ Hydrogen state, given by

$$\varphi(r) = \frac{\pi}{(2\pi a_B)^{3/2}} \left(1 - \frac{r}{2a_B}\right) e^{-r/2a_B} .$$

Put your final expressions in the form, $m_e c^2 \alpha^n$.

2) (20 points) Hyperfine Interaction
Determine the hyperfine shift of the $1s$ state for deuterium atom (an isotope of hydrogen) which has nuclear spin-1.

3) (25 points) Exchange Energy
For two electrons having spatial wave functions $\varphi_a(\vec{r})$ and $\varphi_b(\vec{r})$, give the explicit expression for their exchange energy K_{ab}. Using this expression prove that always $K_{ab} > 0$.
Hint: One way is to make use of Fourier transform properties.

4) (25 points) Spectroscopic Notation and Hund's Rules
Find all possible terms in LS notation for oxygen ($1s^2 2s^2 2p^4$) and fluorine ($1s^2 2s^2 2p^5$). For either case identify the term for the ground state.

Formula Reminder:
$H_{SO} = \lambda \vec{\ell} \cdot \vec{s}$; $H_R = -\frac{p^4}{8m_e^3 c^2}$; $H_D = \frac{e^2 \hbar^2}{8 m_e^2 c^2 \epsilon_0} \delta(\vec{r})$
$H_{hf} = A \left[\frac{3}{8\pi r^3} \left(\vec{\ell} \cdot \vec{I}\right) + \frac{3}{8\pi r^3} \left(\frac{3(\vec{s}\cdot\vec{r})(\vec{I}\cdot\vec{r})}{r^2} - (\vec{s} \cdot \vec{I}) \right) + \left(\vec{s} \cdot \vec{I}\right) \delta(\vec{r}) \right]$
$\langle \frac{1}{r} \rangle_{n\ell} = \frac{Z}{a_B n^2}$; $\langle \frac{1}{r^2} \rangle_{n\ell} = \frac{Z^2}{a_B^2 n^3 (\ell + 1/2)}$; $\langle \frac{1}{r^3} \rangle_{n\ell} = \frac{Z^3}{a_B^3 n^3 (\ell+1)(\ell+1/2)\ell}$
$a_B = \frac{\hbar}{m_e c \alpha}$; $\frac{e^2}{4\pi\epsilon_0} = \alpha \hbar c$