1) From HW-3, solution repeated below

P29-15 The current is found from Eq. 29-5,

$$i = \int \vec{\mathbf{j}} \cdot d\vec{\mathbf{A}},$$

where the region of integration is over a spherical shell concentric with the two conducting shells but between them. The current density is given by Eq. 29-10,

$$\vec{\mathbf{j}} = \vec{\mathbf{E}}/\rho,$$

and we will have an electric field which is perpendicular to the spherical shell. Consequently,

$$i = \frac{1}{\rho} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\rho} \int E\, dA$$

By symmetry we expect the electric field to have the same magnitude anywhere on a spherical shell which is concentric with the two conducting shells, so we can bring it out of the integral sign, and then

$$i = \frac{1}{\rho} E \int dA = \frac{4\pi r^2 E}{\rho},$$

where E is the magnitude of the electric field on the shell, which has radius r such that $b > r > a$.

The above expression can be inverted to give the electric field as a function of radial distance, since the current is a constant in the above expression. Then $E = i\rho/4\pi r^2$ The potential is given by

$$\Delta V = -\int_b^a \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}},$$

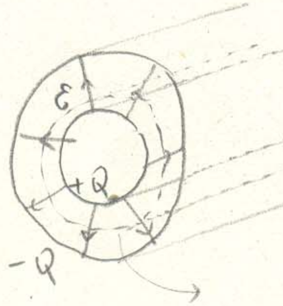
we will integrate along a radial line, which is parallel to the electric field, so

$$\begin{aligned} \Delta V &= -\int_b^a E\, dr, \\ &= -\int_b^a \frac{i\rho}{4\pi r^2}\, dr, \\ &= -\frac{i\rho}{4\pi} \int_b^a \frac{dr}{r}, \\ &= \frac{i\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right). \end{aligned}$$

We divide this expression by the current to get the resistance. Then

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

2] Let's obtain the capacitance per unit length of a full cylindrical geometry (this was done in class)



$$\underbrace{\oint_S \vec{D}.\hat{n}da = Q}$$

only flux through lateral area $\Rightarrow 2\pi r \ell D_r = Q$

$$D_r = \frac{Q}{\ell}\cdot\frac{1}{2\pi r} \Rightarrow E_r = \frac{D_r}{\varepsilon} = \frac{Q}{\ell}\frac{1}{2\pi\varepsilon r}$$

$$\Delta V = V(r=a) - V(r=b) = -\int_b^a \vec{E}.\vec{dl}$$

$$\Rightarrow \Delta V = \frac{Q/\ell}{2\pi\varepsilon}\ln(b/a)$$

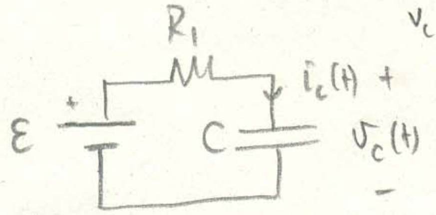
So, for a full cylinder, the capacitance per unit length is $C_{cyl} = \frac{Q/\ell}{\Delta V} = \frac{2\pi\varepsilon}{\ln(b/a)}$

In our case we have two quarter cylinder with ε_1 & ε_2 in parallel

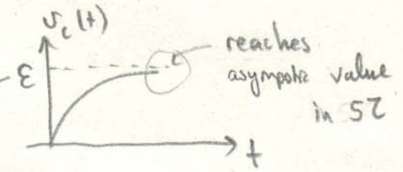
$$\therefore \quad C = \frac{\pi}{2\ln(b/a)}(\varepsilon_1 + \varepsilon_2)$$

3] Charging: $0 < t < 10ms$

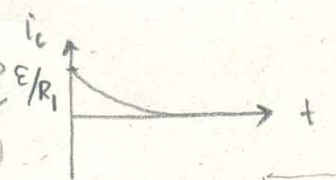
$v_c(0) = 0$



$\tau = R_1 C$



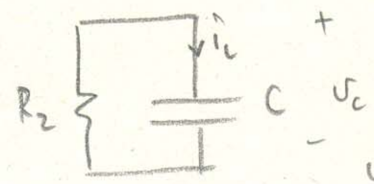
5V



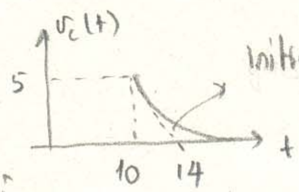
$R_1 = 1K\Omega$

Discharging: $t > 10ms$

$\tau = R_2 C$



$v_c(10) = 5V$

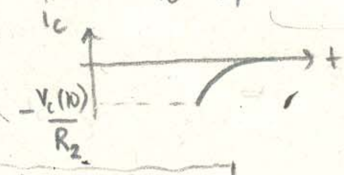


Initial slope gives $-\frac{V_c(10)}{\tau}$

$\Rightarrow \tau = 4ms$

$\Rightarrow C = 2\mu F$

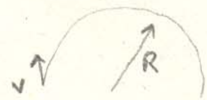
$R_2 = 2K\Omega$



$$\boxed{\varepsilon = 5V,\ R_1 = 1K\Omega,\ R_2 = 2K\Omega,\ C = 2\mu F}$$

4] This is the cyclotron motion under the radial magnetic force $F_M = qvB = m\,a$ uniform circular motion

$$a = \frac{v^2}{R}$$



$$\Rightarrow \qquad R = \frac{mv}{qB}$$

$$x = 2R = \frac{2mv}{qB}, \qquad v = ?$$

A charge gains a kinetic energy $q\Delta V$ from a potential difference ΔV

$$K = \frac{1}{2}mv^2 = q\Delta V \qquad \Rightarrow \qquad v = \sqrt{\frac{2q\Delta V}{m}}$$

$$\Rightarrow \quad x = \frac{2m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{2\sqrt{2}}{B}\sqrt{\frac{\Delta V m}{q}}$$

$$\boxed{m = \frac{B^2 q x^2}{8\Delta V}}$$