PHYS-112 Midterm-I Solutions

8 March 2007

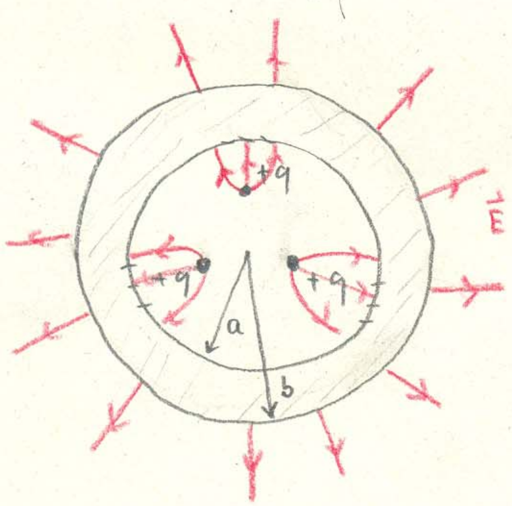
1] a) $\sigma = \varepsilon_0 E = 8.85 \cdot 10^{-12} \cdot (-150) \cong -1.3 \cdot 10^{-9}\ C/m^2$

b) Total charge $\quad q = \sigma\, 4\pi R_{Earth}^2 = -1.3 \cdot 10^{-9}\, 4\pi \cdot (6.4 \cdot 10^6)^2 \simeq -700 \cdot 10^3\ C$

$$N = \frac{q}{e} \cong 4 \cdot 10^{24}$$

c) The excess negative electric charge should be compensated by the +ve ions spread over the atmosphere. Therefore outside the atmosphere there will be (almost) no E-field. Thunderstorms are caused by the local build up of excess charge.

2] a)



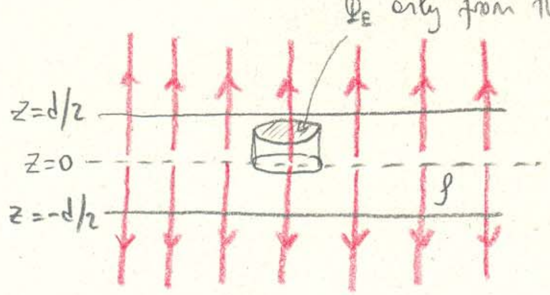
b) $V(b) = V(a)$ ... conductor

For $r > b$, the problem is equivalent to $+3q$ charge at center

$$\therefore\ V(b) = \frac{3q}{4\pi\varepsilon_0}\frac{1}{b}$$

$$\Rightarrow\ V(a) = \frac{3q}{4\pi\varepsilon_0}\frac{1}{b}$$

3]



← $E(z=0) = 0$ due to symmetry

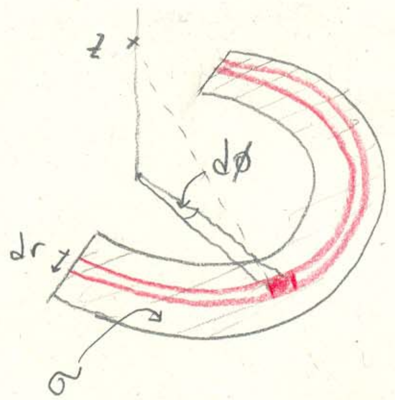
Choosing the pillbox shaped Gaussian surfaces with one face at $z=0$ plane, we obtain

$$\underbrace{\oint_S \vec{E}\cdot\hat{n}\,da}_{EA} = \begin{cases} \dfrac{A\rho z}{\varepsilon_0}, & z \le d/2 \\[2ex] \dfrac{A\rho d}{2\varepsilon_0}, & z > d/2 \end{cases}$$

$$\Rightarrow \quad \vec{E}(z) = \begin{cases} \hat{k}\,\dfrac{\rho d}{2\varepsilon_0} & , \quad z > d/2 \\ \hat{k}\,\dfrac{\rho z}{\varepsilon_0} & , \quad |z| < d/2 \\ -\hat{k}\,\dfrac{\rho d}{2\varepsilon_0} & , \quad z < -d/2 \end{cases}$$

4]

a)



$$V(z) = \frac{1}{4\pi\varepsilon_0} \int_a^b \int_0^\pi \frac{\sigma\, r\,dr\,d\phi}{\sqrt{z^2+r^2}}$$

$$V(z) = \frac{\pi\sigma}{4\pi\varepsilon_0} \int_a^b \frac{r\,dr}{\sqrt{z^2+r^2}}$$

Let $u \equiv z^2 + r^2$; $du = 2r\,dr$

$$\Rightarrow \quad V(z) = \frac{\sigma}{4\varepsilon_0} \cdot \underbrace{\int_{z^2+a^2}^{z^2+b^2} u^{-1/2}\,\frac{du}{2}}_{\sqrt{z^2+b^2}-\sqrt{z^2+a^2}}$$

$$\Rightarrow \quad V(z) = \frac{\sigma}{4\varepsilon_0}\left[\sqrt{z^2+b^2} - \sqrt{z^2+a^2}\right]$$

b) $$E_z(z) = -\frac{\partial V(z)}{\partial z}$$

$$E_z(z) = \frac{\sigma z}{4\varepsilon_0}\left[\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{\sqrt{z^2+b^2}}\right]$$

Note that, $\vec{E}$-field has other components as well.