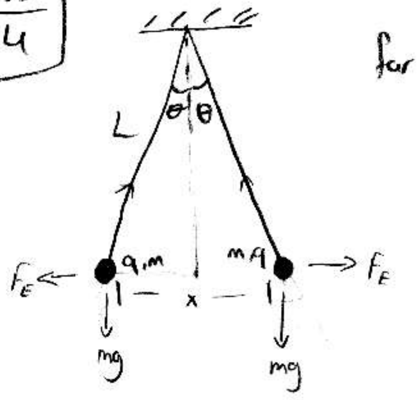
Solutions to HW 1

Ch 25 Pr 4



for small θ $\tan\theta \approx \sin\theta$, you can see this by Taylor expansion.

$$F_E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{x^2} \Leftarrow \boxed{\begin{array}{l}\dfrac{x}{2L} = \sin\theta \\ mg\tan\theta \simeq mg\sin\theta = F_E\end{array}}$$

$$\Rightarrow mg\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}$$

$$mg\frac{x}{2L} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \Rightarrow x^3 = \frac{1}{2\pi\varepsilon_0}\frac{q^2L}{mg}$$

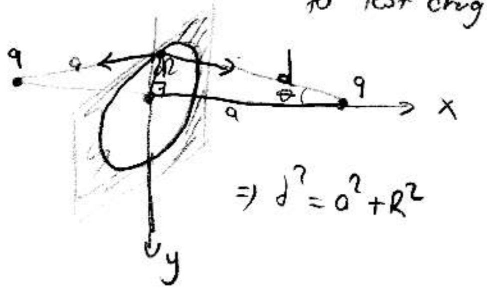
$$x = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2L}{mg}\right)^{1/3}$$

b-)
$L = 1{,}12\,m$
$m = 11{,}2\ 10^{-3}\,kg$
$x = 4{,}7\times10^{-2}\,m$
$g = 9{,}8\ m/s^2$
$\varepsilon_0 = 8{,}85\times10^{-12}\,F/m$
$= 8{,}85\times10^{-12}\,C^2/Nm^2$

$$\pm\sqrt{\frac{2\pi\varepsilon_0 x^3 mg}{L}} = q$$

$$\boxed{q = \pm\, 2{,}38\times10^{-8}\,C}$$

Ch 25 Pr 11



for test charge q_0. $\Rightarrow F = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{(a^2+R^2)}$

$\Rightarrow 2F\sin\theta = F_y$ is equal total force.

$\Rightarrow d^2 = a^2 + R^2$

$$\Rightarrow 2k\frac{q_0 q}{(a^2+R^2)}\cdot\frac{R}{(a^2+R^2)^{1/2}} = F_y$$

$\Rightarrow \frac{d}{dR}F_y(R) = 0$ gives us the max value; $F_y'(R) = \frac{q q_0}{2\pi\varepsilon_0}\left(\frac{(a^2+R^2)^{3/2} - 3R^2(a^2+R^2)^{1/2}}{(a^2+R^2)^3}\right) = 0$

$\Rightarrow$ After some algebra;

$$\boxed{R = \pm\frac{a}{\sqrt{2}}}$$

Ch 26

Pr 5

⇒ for this question, we have to make, a little bit observation:

- In case of dipole, for $x \gg d$, (c.f. pg. 592 of text book) Electric field is given by:

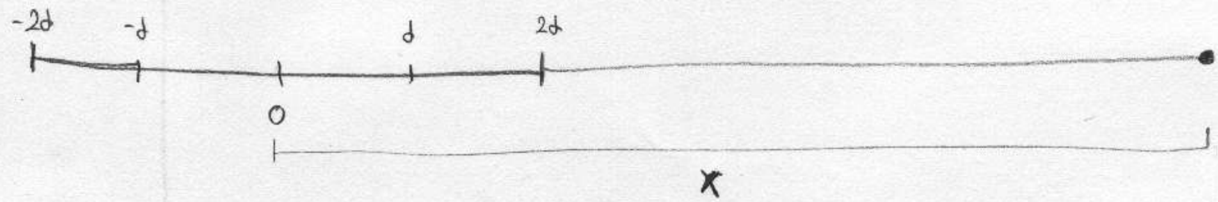
$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3} \Rightarrow \text{depends on } x^3$$

- In case of quadrupole, for $x \gg d$, (you can check it from 4th problem) E field is given by

$$E = \frac{3Q}{4\pi\varepsilon_0 x^4} \Rightarrow \text{depends on } x^4$$

⇒ In same fashion we can continue to construct octopole (x^{-5} dependent) and 16-pole

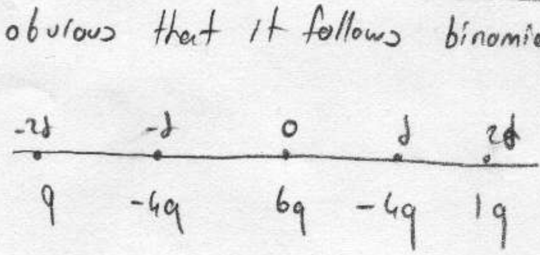
x^6 dependent



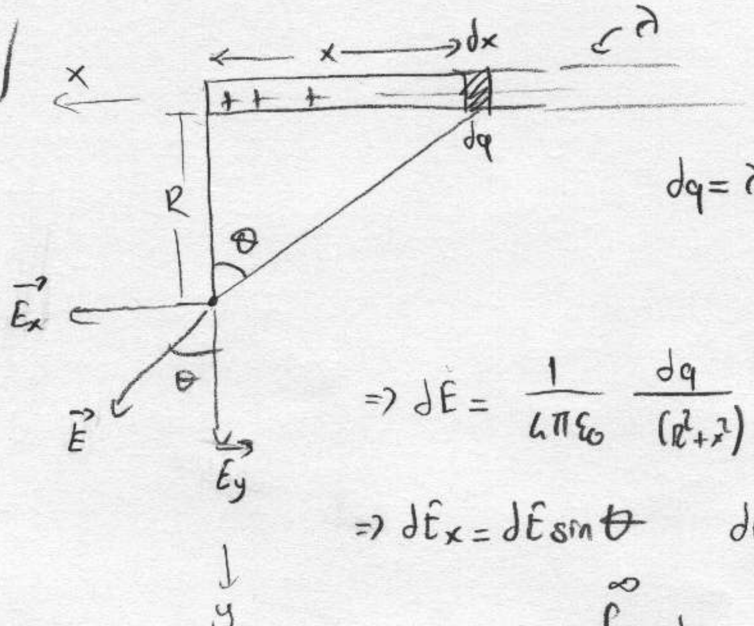
⇒ As a final observation:

monopole;	±1q				
dipole :	+1q	-1q			
quadrupole :	+1q	-2q	+1q		
octopole :	+1q	-3q	+3q	-1q	
16-pole :	+1q	-4q	+6q	-4q	+1q

It is obvious that it follows binomial expansion. So arrangement of charges are;



Chapter 26
Problem 6



$dq = \lambda dx$

$$\Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+x^2)}$$

$$\Rightarrow dE_x = dE\sin\theta \qquad dE_y = dE\cos\theta$$

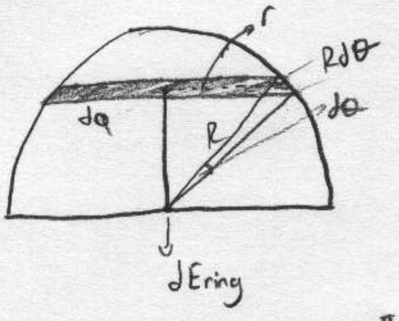
$$\Rightarrow E_x = \frac{1}{4\pi\varepsilon_0}\int_0^\infty \frac{x\,dx}{(x^2+R^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R}$$

$$\Rightarrow E_y = \frac{1}{4\pi\varepsilon_0}\int_0^\infty \frac{R\,dx}{(x^2+R^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{R}$$

$\Rightarrow \tan\theta = \dfrac{E_x}{E_y}$ $\Rightarrow$ Here we find $\tan^{-1} 1 = \boxed{\theta = 45^\circ}$

Thus we see that θ is independent of R

Chapter 26
Problem 8



$\Rightarrow r = R\cos\theta$ $\qquad \sigma = \dfrac{q}{2\pi R^2}$

$$\rightarrow dq = \sigma \cdot (2\pi R\cdot\cos\theta)(R\cdot d\theta)$$

$$\rightarrow \sin\theta\cdot\cos\theta = \tfrac{1}{2}\sin 2\theta$$

$$\Rightarrow dE_{ring} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi R^2\cos\theta\sin\theta d\theta}{R^2}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}\int_{\theta=0}^{\theta=\pi/2}\frac{1}{2}\sin 2\theta d\theta = \frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{4}\cos 2\theta\right)\Big|_0^{\pi/2} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2}\right)$$

$$\Rightarrow E = \frac{q}{8\pi\varepsilon_0 R^2}$$